All questions must be answered. Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

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MONETARY ECONOMICS: MACRO ASPECTS
SOLUTIONS TO AUGUST 16, 2013 EXAM

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) A higher degree of substitutability between consumption goods makes aggregate inflation more costly in a New-Keynesian model with Calvo price setting.

A True. With more substitutability, any relative price change creates more dispersion in demand of various goods, which is welfare deteriorating. Under Calvo price setting, higher inflation creates larger changes in relative prices. Hence, higher inflation is more costly with more substitutable goods.

(ii) In the simple Poole model for the choice of operating procedures for monetary policy (where output stability is all that matters), more exogenous volatility in the goods market favors the adoption of an interest-rate operating procedure.

A False. Any shock on the goods market moves output one for one under an interest-rate operating procedure. On the other hand, under a money-supply operating procedure, the endogenous response of the interest rate is (partially) output stabilizing. E.g., a positive increase in goods demand and output are accompanied by an increase in money demand and an increase in the interest rate, which dampens the expansion. Hence, more exogenous volatility in the goods market will *ceteris paribus* tend to disfavor an interest-rate operating procedure.

(iii) In the Barro and Gordon inflation bias model, the optimal commitment rule involves more stable inflation compared with discretionary policymaking.

A False. In the Barro and Gordon model, the inefficiency arising under time-consistent policymaking is the inflation bias arising due to a too high desired output level on the policymaker's behalf. Stabilization of shocks is carried out efficiently, and optimally spreads out the effect of supply shocks on output and inflation. Under commitment, the policymaker is able to avoid the inflation bias, but, of course, stabilizes supply shocks efficiently.

QUESTION 2:

Inflation targeting and noisy data

Consider the following log-linear New-Keynesian model of a closed economy:

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left(\widehat{i}_t - \mathrm{E}_t\pi_{t+1}\right), \qquad \sigma > 0, \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

where x_t is the output gap (output's deviation from the flexible-price output), $\widehat{i}_t$ is the nominal interest rate's deviation from steady state, π_t is goods price inflation. E_t is the rational expectations operator conditional upon all information up to and including period t. Inflation is assumed to be observed with some error such that

$$\pi_t^o = \pi_t + e_t, \tag{3}$$

where π_t^o denotes observed inflation, and e_t is a mean-zero, serially uncorrelated shock accounting for noise in the data. It is assumed that the central bank sets the nominal interest rate according to a simple rule:

$$\widehat{i}_t = \phi \pi_t^o, \qquad \phi > 1. \tag{4}$$

(i) Describe in detail the micro foundations behind (1) and (2).

A Equation (1) is the dynamic IS curve, which is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, $\widehat{i}_t - \mathrm{E}_t\{\pi_{t+1}\}$, make consumers increase future consumption relative to current. Equation (2), the New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be in effect for some periods.

(ii) Solve for x_t and π_t [Hint: Conjecture that solutions are linear functions of e_t.], and explain how the policy-rule parameter ϕ affects output gap and inflation fluctuations.

A Substitute the definition of observable inflation, (3), into the policy rule, (4), and insert the resulting expression

$$\widehat{i}_t = \phi(\pi_t + e_t)$$

into (1):

$$x_t = \mathrm{E}_t x_{t+1} - \sigma^{-1}\left[\phi(\pi_t + e_t) - \mathrm{E}_t \pi_{t+1}\right]. \tag{*}$$

Now conjecture that

$$\pi_t = -A_\pi e_t, \qquad x_t = -A_x e_t,$$

implying

$$\begin{aligned} \mathrm{E}_t \pi_{t+1} &= -A_\pi \mathrm{E}_t e_{t+1} = 0, \\ \mathrm{E}_t x_{t+1} &= -A_x \mathrm{E}_t e_{t+1} = 0. \end{aligned}$$

Use the conjecture and their expectations in (*) and (2) to get

$$\begin{aligned} -A_x e_t &= -\sigma^{-1}\phi(-A_\pi e_t + e_t), \\ -A_\pi e_t &= -\kappa A_x e_t. \end{aligned}$$

This verifies the form of the conjectures and identify the unknown coefficients as

$$\begin{aligned} A_x &= \sigma^{-1}\phi(1 - A_\pi), \\ A_\pi &= \kappa A_x. \end{aligned}$$

From this we readily recover

$$\begin{aligned} A_x &= \sigma^{-1}\phi(1 - \kappa A_x) \\ A_x &= \frac{\sigma^{-1}\phi}{1 + \sigma^{-1}\phi\kappa} \end{aligned}$$

and

$$A_\pi = \frac{\sigma^{-1}\phi\kappa}{1 + \sigma^{-1}\phi\kappa}$$

Hence, the solutions for the output gap and inflation are

$$\begin{aligned} x_t &= -\frac{\sigma^{-1}\phi}{1 + \sigma^{-1}\phi\kappa} e_t \\ \pi_t &= -\frac{\sigma^{-1}\phi\kappa}{1 + \sigma^{-1}\phi\kappa} e_t \end{aligned}$$

(iii) Does this model lend support to the view that a central bank should respond strongly towards observed inflation? Why/Why not?

A The solutions show that the response coefficient on observed inflation plays a crucial role for output and inflation fluctuations. Indeed, the higher is ϕ, the higher are the respective impacts of the measurement error on the macroeconomy. For example, in the special case where the central bank responds infinitely strong to observed inflation, $\phi \to \infty$, the shock e_t is transmitted fully onto inflation. ($\lim_{\phi \to \infty} -\frac{\sigma^{-1}\phi\kappa}{1+\sigma^{-1}\phi\kappa} = -1$.) Hence, this model does not lend support to a strong response towards observed inflation. Intuitively, letting policy be strongly guided by noise, creates noise in the economy.

QUESTION 3:

Monetary shocks and imperfect information

Consider a version of Lucas' flex-price model where individuals live on isolated islands, and after each period are randomly relocated to another island. Letting superscript "i" denote island variables, and no superscript denote economy-wide average variables, four central equations describing the economy are

$$Y_t^i = \left(N_t^i\right)^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

$$C_t^i = Y_t^i, \tag{2}$$

$$u_{1-N}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) = \left[(1-\alpha)\frac{Y_t^i}{N_t^i}\right] u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right), \tag{3}$$

$$\begin{aligned} u_C\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right) &= u_{M/P}\left(C_t^i, M_t^i/P_t^i, 1-N_t^i\right), \qquad 0 < \beta < 1, \\ &\quad +\beta \mathrm{E}^i u_C\left(C_{t+1}, M_{t+1}/P_{t+1}, 1-N_{t+1}\right), \end{aligned} \tag{4}$$

where Y_t is output in period t, N_t is employment, C_t is consumption, M_t is the nominal money supply at the end of the period, and P_t is the price level. The function u is increasing and concave in all arguments, and u_j denotes the partial derivative of u with respect to variable j. E^i denotes expectations conditional on local information.

(i) Discuss equations (1)–(4) and explain how a change in the real money supply can have real effects in the model.

A (1) is the local production function; (2) is local market clearing; (3) is the condition for optimal labor supply; (4) is the condition for optimal money demand. In this money-in-the-utility function set up, a change in the real money stock can have real effects by affecting labor supply through (3). For example, if the marginal utility of consumption is increasing in the real money stock, the return from work increases and labor supply goes up when the real money stock rises. If the utility function is separable in real money, there will be no effects at all.

The stochastic process for the log of nominal money on island i, m_t^i, is given by

$$m_t^i = \gamma m_{t-1}^i + u_t + u_t^i, \qquad 0 < \gamma < 1, \tag{5}$$

where u_t^i is an island-specific shock with mean zero and variance σ_i^2, and u_t is an aggregate shock with mean zero and variance σ_u^2. The shocks u_t and u_t^i are assumed independent, and the informational assumptions are as follows: On island i, variables m_t^i and γm_{t-1}^i are known. The variables u_t and u_t^i, cannot be observed; their sum, however, can be inferred perfectly.

(ii) Discuss how such imperfect information about u_t and u_t^i can affect equilibrium real behavior as a result of a change in aggregate nominal money, i.e., a change in u_t.

A If agents know that a shock is global, they know that all prices on all Islands will increase proportionally leaving real money unchanged. No agent would change behavior. If the shock is known to be local, on the other hand, the agents know that all prices will not adjust proportionally, and the real value of carrying money increases. Labor supply will then be affected as discussed in (i). When there is imperfect information, agents will rationally guess that the shock has some global and some local component leading to some response.

(iii) Derive $\mathrm{E}^i\left[u_t | u_t + u_t^i\right]$ under the assumption that expectations about u_t are formed by use of a linear least squares projection. (Hint: agents make an estimate of u_t, which is a linear function of what is observed, $\widehat{u}_t = \kappa\left(u_t + u_t^i\right)$, where κ is the estimation coefficient minimizing the squared forecast error.) Discuss how σ_i^2 and σ_u^2 affect expectations about u_t and thereby the magnitude of real effects of nominal shocks.

A Using the hint, we derive κ as the solution to

$$\min_{\kappa} \ \mathrm{E}\left[\widehat{u}_t - u_t\right]^2,$$

$$\min_{\kappa} \ \mathrm{E}\left[\kappa\left(u_t + u_t^i\right) - u_t\right]^2,$$

$$\min_{\kappa}\ \mathrm{E}\left[\kappa^2\left(u_t+u_t^i\right)^2+u_t^2-2\kappa\left(u_t+u_t^i\right)u_t\right],$$

As shocks are independent and have zero means, this becomes

$$\min_{\kappa}\ \left(\kappa^2\mathrm{Var}\left[u_t+u_t^i\right]+\mathrm{Var}\left[u_t\right]-2\kappa\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]\right).$$

The first-order condition is

$$\kappa\mathrm{Var}\left[u_t+u_t^i\right]-\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]=0,$$

leading to

$$\kappa=\frac{\mathrm{Cov}\left[\left(u_t+u_t^i\right)u_t\right]}{\mathrm{Var}\left[u_t+u_t^i\right]}=\frac{\mathrm{Var}\left[u_t\right]}{\mathrm{Var}\left[u_t\right]+\mathrm{Var}\left[u_t^i\right]}=\frac{\sigma_u^2}{\sigma_u^2+\sigma_i^2}<1,$$

and thus

$$\mathrm{E}^i\left[u_t|u_t+u_t^i\right]=\kappa\left(u_t+u_t^i\right)=\frac{\sigma_u^2}{\sigma_u^2+\sigma_i^2}\left(u_t+u_t^i\right).$$

One notes that the higher is σ_u^2 relative to σ_i^2 the more likely it is that the change in $u_t+u_t^i$ are caused by changes in u_t rather than in u_t^i. In consequence, κ increases. In such a case, the effect of an aggregate monetary shock will be smaller.